TRINITY COLLEGE

Semester One Examination, 2023

Question/Answer booklet

MATHEMATICS SPECIALIST UNIT 1

If required by your examination administrator, please place your student identification label in this box

Section One: Calculator-free

WA student number: In figures | | | | | | | | | |

In words _____

_____

Your name _____

Time allowed for this section

Reading time before commencing work: five minutes
Working time: fifty minutes

Materials required/recommended for this section

To be provided by the supervisor

This Question/Answer booklet
Formula sheet

To be provided by the candidate

Standard items: pens (blue/black preferred), pencils (including coloured), sharpener, correction fluid/tape, eraser, ruler, highlighters

Special items: nil

Important note to candidates

No other items may be taken into the examination room. It is **your** responsibility to ensure that you do not have any unauthorised material. If you have any unauthorised material with you, hand it to the supervisor **before** reading any further.

Structure of this paper

Section	Number of questions available	Number of questions to be answered	Working time (minutes)	Marks available	Percentage of examination
Section One: Calculator-free	7	7	50	49	35
Section Two: Calculator-assumed	12	12	100	94	65
				Total	100

Instructions to candidates

1. The rules for the conduct of Trinity College examinations are detailed in the *Instructions to Candidates* distributed to students prior to the examinations. Sitting this examination implies that you agree to abide by these rules.

2. Write your answers in this Question/Answer booklet preferably using a blue/black pen. Do not use erasable or gel pens.

3. You must be careful to confine your answers to the specific question asked and to follow any instructions that are specific to a particular question.

4. Show all your working clearly. Your working should be in sufficient detail to allow your answers to be checked readily and for marks to be awarded for reasoning. Incorrect answers given without supporting reasoning cannot be allocated any marks. For any question or part question worth more than two marks, valid working or justification is required to receive full marks. If you repeat any question, ensure that you cancel the answer you do not wish to have marked.

5. It is recommended that you do not use pencil, except in diagrams.

6. Supplementary pages for planning/continuing your answers to questions are provided at the end of this Question/Answer booklet. If you use these pages to continue an answer, indicate at the original answer where the answer is continued, i.e. give the page number.

7. The Formula sheet is not to be handed in with your Question/Answer booklet.

See next page

Section One: Calculator-free **35% (49 Marks)**

This section has **seven** questions. Answer **all** questions. Write your answers in the spaces provided.

Working time: 50 minutes.

Question 1 **(6 marks)**

Relative to an origin O, the position vectors of points K, L and M are $\begin{pmatrix}5\\1\end{pmatrix}, \begin{pmatrix}-1\\-7\end{pmatrix}$ and $\begin{pmatrix}-3\\-1\end{pmatrix}$ respectively.

(a) Determine $\overrightarrow{LK}$. (1 mark)

(b) Determine the magnitude of $\overrightarrow{LK}$. (1 mark)

(c) Determine a vector in the same direction as $\overrightarrow{LM}$ that has the same magnitude as $\overrightarrow{LK}$. (2 marks)

(d) If KM is a diagonal of parallelogram $KLMN$, determine the position vector of vertex N. (2 marks)

Question 2 **(6 marks)**

A student, mindful of the fact that a square, rectangle and rhombus are special cases of a parallelogram, wrote the following true statement:

If a quadrilateral is a parallelogram, then opposite sides in the quadrilateral are equal in length.

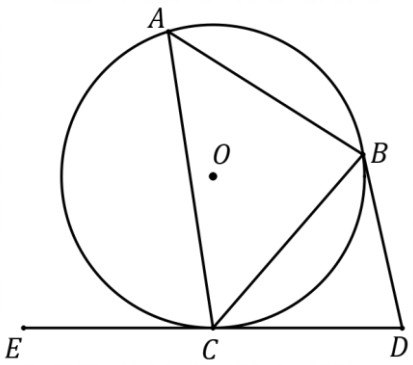
(a) Write the inverse statement and state whether it is true. (2 marks)

(b) Write the contrapositive statement and state whether it is true. (2 marks)

(c) Write the converse statement and state whether it is true. (2 marks)

Question 3 **(7 marks)**

In the diagram, DE is tangential to the circle with centre O at C and the vertices of triangle ABC lie on the circle as shown.



(a) Prove the angle in the alternate segment theorem, that $\angle BCD = \angle BAC$. (4 marks)

(b) If $\angle CDB = \alpha, \angle CBD = \beta$ and $CA = CB$, prove that $\angle ACB = 2\alpha + 2\beta - 180°$. (3 marks)

Question 4 **(9 marks)**

Consider the vectors $\underset{\sim}{a} = \begin{pmatrix}3\\1\end{pmatrix}$, $\underset{\sim}{b} = \begin{pmatrix}5\\-2\end{pmatrix}$ and $\underset{\sim}{c} = \begin{pmatrix}9\\-8\end{pmatrix}$. Determine

(a) a unit vector in the same direction as $3\underset{\sim}{a} - \underset{\sim}{c}$. (2 marks)

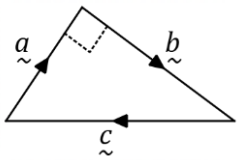
(b) vectors $\underset{\sim}{r}$ and $\underset{\sim}{s}$ given that $\underset{\sim}{a} = \underset{\sim}{r} + \underset{\sim}{s}$ and $\underset{\sim}{b} = 2\underset{\sim}{r} + \underset{\sim}{s}$. (3 marks)

(c) the value of the constants λ and μ when $\lambda\underset{\sim}{a} + \mu\underset{\sim}{b} = \underset{\sim}{c}$. (4 marks)

Question 5 **(7 marks)**

(a) Consider parallelogram $OABC$, so that $\overrightarrow{OA} = \underset{\sim}{a}$ and $\overrightarrow{OC} = \underset{\sim}{c}$. Use a vector method to show that when diagonals $\overrightarrow{OB}$ and $\overrightarrow{AC}$ are perpendicular, then the parallelogram is a rhombus. (4 marks)

(b) Vectors $\underset{\sim}{a}, \underset{\sim}{b}$ and $\underset{\sim}{c}$ are chosen as in the diagram shown, so that $\underset{\sim}{a}$ and $\underset{\sim}{b}$ are perpendicular.
Use a vector method to prove Pythagoras' theorem.



(3 marks)

Question 6 **(7 marks)**

(a) The letters of the word MATAMATA are arranged randomly in a line. Determine

(i) the number of different ways this can be done. (2 marks)

(ii) the number of arrangements in which all the A's are grouped together. (2 marks)

(b) A class of 13 children together picked up 75 pieces of litter. Prove that at least two children picked up the same number of pieces of litter. (3 marks)

See next page

Question 7 **(7 marks)**

(a) Vectors $\underset{\sim}{r}$ and $\underset{\sim}{s}$ have magnitudes 4 and 3 respectively, and $\underset{\sim}{r} \cdot \underset{\sim}{s} = -2$. Evaluate

(i) $\left(2\underset{\sim}{r}\right) \cdot \left(\frac{1}{4}\underset{\sim}{s}\right)$. (1 mark)

(ii) $\left(3\underset{\sim}{r} - \underset{\sim}{s}\right) \cdot \left(\underset{\sim}{r} - 4\underset{\sim}{s}\right)$. (3 marks)

(b) The vector projection of $\begin{pmatrix}3\\t\end{pmatrix}$ on $\begin{pmatrix}-3\\1\end{pmatrix}$ is $\begin{pmatrix}6\\-2\end{pmatrix}$. Determine the value of the constant t.

(3 marks)

End of questions

Supplementary page

Question number: _____

Supplementary page

Question number: _____